UNIT #(01) : MOTION & FORCE

Distance: Length of path followed

Displacement: Shortest Distance.

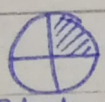
- $S > |\vec{d}|$ (or) $S/|\vec{d}| > 1$ for NON-LINEAR PATH
- $S = |\vec{d}|$ (or) $S/|\vec{d}| = 1$ for UNI-DIRECTIONAL LINEAR PATH
- $S < |\vec{d}|$ (or) $S/|\vec{d}| < 1$ is IMPOSSIBLE IN ANY SCENARIO

IN A CIRCLE, the length of the path is considered to be the circumference which is $2\pi R$. So diff. lengths and hence distances can be given as,

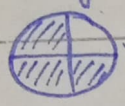
If body covers $1/2$ the circle $\Rightarrow$ then

① $\frac{2\pi R}{2} = \pi R = S$

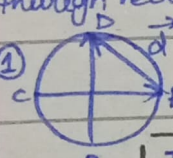
If body covers $1/4$ the circle $\Rightarrow$ then

②  $\frac{2\pi R}{4} = \frac{\pi R}{2} = S$

If body covers $3/4$ the circle $\Rightarrow$ then

③  $3 \times \frac{2\pi R}{4} = \frac{3\pi R}{2} = S$

IN A CIRCLE, The displacement still remains shortest distance and is mostly calculated through vector calculations...

①  Shortest Distance hence displacement still appears to be $A \to D = \vec{d}$

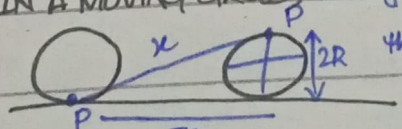
In (1) and (2) magnitude will be calculated as,

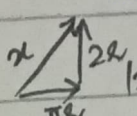
$$\sqrt{(A)^2+(B)^2-2(A)(B)\cos\theta}$$

In (3) magnitude will be,

$$\sqrt{(A)^2+(B)^2+2(A)(B)\cos\theta}$$

IN A MOVING CIRCLE, Having half rotation, the displacement will be given by 'x' of the point P. And distance covered due to half rotation is $2\pi R/2 = \pi R$

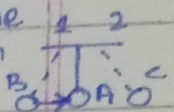




$$|\vec{x}| = \sqrt{(\pi R)^2+(2R)^2+2(\pi R)(2R)\cos\theta}$$

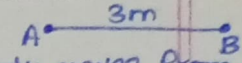
$$\vec{x}^2 = (\pi R)^2 + (2R)^2$$

★ Direction of displacement is always along the direction of motion except in SHM.



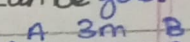
★ In SHM, the displacement is with respect to a mean position. means: ball moving in situation 1 is having motion A → B while displacement B → A same for situation #(02).

★ Distance of a moving body can never be negative and can never be reduced.

Reason: 

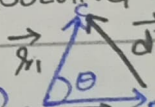
If a body moves from A to B and comes back to A then distance is not negative but becomes 3+3 = 6m. And with time distance cannot be reduced whatsoever.

★ Displacement of a moving body can be zero.

Reason: 

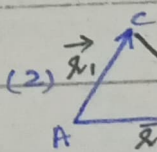
Displacement depends on the initial and final points and hence if the initial and final point is same i.e: covers 3m A to B and then -3m B to A then displacement at A will be = 0.

SOLVING VECTORS:



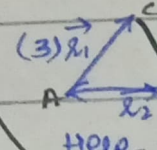
B to C

Here $R_1^2 = \vec{R_2}^2 + \vec{d}^2$



C to B

Here $\vec{R_2}^2 = \vec{R_1}^2 + \vec{d}^2$



C to B

Here $\vec{d}^2 = \vec{R_1}^2 + \vec{R_2}$

MECHANICS

STATIC
Forces acting on body at rest under equilibrium

DYNAMIC
Forces acting on bodies that cause motion

Rest/Motion → Both are states of motion

KINEMATICS
Without taking into account factor causing motion i-e: force.

KINETICS
Considering the cause of motion i-e: force

TRANSLATIONAL MOTION → Motion in a line such that each particle of body has same velocity as the body itself.

- **RECTILINEAR** → Along an absolute straight line.
- **CURVILINEAR** → Along a curved line.

CIRCULAR MOTION: Axis spinning + moving along center of circle.

ROTATORY MOTION: involves axis spinning

TRAJECTORY → Simply means the path of motion.

OSCILLATORY MOTION: Back & Forth motion about fixed point.

Negative Value of Vector → Quantity which is in the opposite direction of reference.

B O A

★ Agr body move kre O sy A tw magnitude (+ve). Ab A sy wapis O aagai tw kiu k displacement can be negative ab (-ve). Lekin agr ab O sy B jaie gee tw wapis sy (+ve) kiu k actual shortest distance tw yahi bnu raha hai.

WRITING VECTOR FORM

3m east → $3\hat{i}$
5m south → $-5\hat{j}$
4m west → $-4\hat{i}$
6m north → $6\hat{j}$
2m upward → $2\hat{k}$
7m downward → $-7\hat{k}$

(East, West) $\hat{i}$
(South, North) $\hat{j}$
(Up/Down ward) $\hat{k}$

e.g: $X = (2\hat{k} + 3\hat{j})$

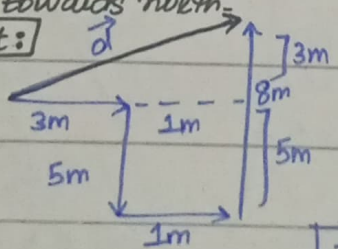
$X = \sqrt{(2)^2 + (3)^2}$

e.g: $Y = (3\hat{i} - 5\hat{j} + 7\hat{k})$

$Y = \sqrt{(3)^2 + (5)^2 + (7)^2}$

MCQ: Car moves 3m east then, 5m south, 1m again east, and in last 8m towards north.

Self-Hint:

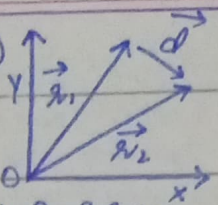


Displacement kre liye iss liye bnai kiu k hamai pta hai initial sy final point jaie gy. "8m" north ko hm ny iss liye "3m" aur "5m" kia kiu k "5m" south wala part nikal jaie tw "3m" bachta. Base pur kraby huye 1m east aur kiu k "4m" bnti...

CIRCLE

Part	Distance	Displacement
1/4th	$S = \frac{1}{4} \times 2\pi r = \pi r/2$	$\sqrt{2}r$
2/4 / 1/2	$S = \frac{1}{2} \times 2\pi r = \pi r$	$2r$
3/4th	$S = \frac{3}{4} \times 2\pi r = 3\pi r/2$	$\sqrt{2}r$ $\sqrt{2}r$
4/4	$S = 1 \times 2\pi r = 2\pi r$	Zero

Velocity: How fast space coordinates (position vector) of a particle relative to a fixed point changes with time.



$\vec{r_1} = x_1\hat{i} + y_1\hat{j}$ $\vec{r_2} = x_2\hat{i} + y_2\hat{j}$

$\vec{d} = \vec{r_2} - \vec{r_1}$

- $\vec{d}$ is independent of origin 'O'.
- Position vectors might change with change in origin.

Speed: Rate at which length of particle changes.

Average Speed & Velocity: are equal for uni-directional motion.

$V_{av} = V_1t_1 + V_2t_2 / t_1 + t_2$ (OR) $V_{av} = d_1 + d_2 / t_1 + t_2$

Cases: (a) Same Time $V_{av} = V_1 + V_2 / 2$ (b) Same distance/displacement $V_{av} = 2V_1V_2 / V_1 + V_2$

Uniform Speed & Velocity: Speed/Velocity and Time remain constant.

Uniform velocity → only position vector is variable.

Variable Velocity: either direction, speed changes or both change.

> In a circle, speed constant, but velocity variable bcz direction is variable

Resultant Velocity: $V_R = V_A + V_B$

Relative Velocity: of A with respect to B = $V_B - V_A$. of B w.r.t A = $V_A - V_B$

Stopping Distance: directly ∝ to initial velocity bcz $V_f = 0$

Relative Velocity (a) For same direction: $V_{rel} = V_1 - V_2$

(b) For opposite direction: $V_{rel} = V_1 - (-V_2)$

CASES FOR SOLUTION OF AVERAGE SPEED & VELOCITY

(a) If velocity & time available:

$$V_{av} = \frac{V_1t_1 + V_2t_2}{t_1 + t_2}$$

(b) If displacement & time:

$$V_{av} = \frac{d_1 + d_2}{t_1 + t_2}$$

(c) If $t_1 = t_2 = t$

$$V_{av} = \frac{V_1 + V_2}{2}$$

$$V_{av} = \frac{V_1 + V_2 \dots V_n}{n}$$

(d) If $S_1 = S_2 = S$

$$V_{av} = \frac{2}{1/V_1 + 1/V_2}$$

$$V_{av} = \frac{n}{1/V_1 + 1/V_2 \dots 1/V_n}$$

(e) If displacement and time both are same:

$V_{av} = V_{instant}$

ACCELERATION: Rate of change of velocity.

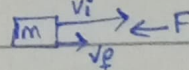
$$\vec{a} = \frac{\Delta\vec{V}}{\Delta t} = \frac{\vec{V_f} - \vec{V_i}}{\Delta t} = \frac{\vec{V_f} + (-\vec{V_i})}{\Delta t}$$

Unit = ms^{-2} Quantity = vector

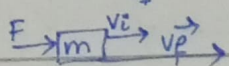
→ Direction of acceleration is always in the direction of Net force / change in velocity.

CASES & POINTS

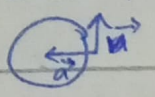
$\vec{v}$ anti-parallel $\vec{a}$: Velocity decreases ⇒



$\vec{v}$ parallel $\vec{a}$: velocity increases ⇒



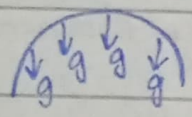
$\vec{v} \perp \vec{a}$: Velocity might change and can even be constant, bcz direction changes and hence acc. has to occur.



★ If velocity is zero, acceleration doesn't have to be zero in case of motion under gravity i-e: at the topmost part.

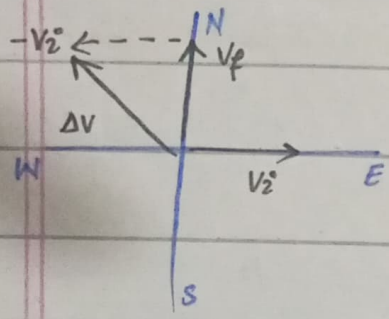
★ If velocity constant → Acceleration Zero

★ Projectile motion → Curve with const. acceleration.



+ve acc	-ve acc.
ΔV = +ve	ΔV = -ve
V → inc.	V → dec.
$V_f > V_i$	$V_f < V_i$

MCQ: A car is moving towards east. after some time it starts moving towards north with same speed. Direction of acceleration is:
(a) East (b) North east
(c) North west (d) South east.



Reason: change in velocity is addition of final velocity and negative of initial velocity.

★ It is possible that acc. decreases yet velocity is increasing bcz $5ms^{-2} \rightarrow 2ms^{-2}$ still velocity can be increasing but if acc. becomes negative, then obv velocity decreases

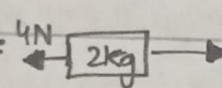
★ If velocity constant Direction changes → Acc. ⊥ Velocity e.g: Circular motion

TYPES OF ACCELERATION

Uniform → constant
Variable → changing velocity in equal interval of time.
Average → $\frac{\text{Total change in V}}{\text{Total change in t}}$
Instantaneous → In such a short interval that $\Delta t \rightarrow 0$

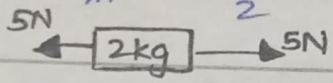
$F = ma$

m = const.	F = const.	a = const.
$\frac{a_1}{a_2} = \frac{F_1}{F_2}$	$\frac{a_1}{a_2} = \frac{m_2}{m_1}$	$\frac{F_1}{F_2} = \frac{m_1}{m_2}$

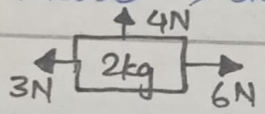
MCQ: 

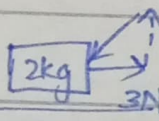
, a = ?

$a = \frac{F_{Net}}{m} = \frac{10 + (-4)}{2} = 3ms^{-2}$



$F_{net} = 0 \rightarrow a = 0$





$F_{net} = \sqrt{(4)^2 + (3)^2}$ (OR)
$= \sqrt{25}$
$= 5N$

$F_{net} = 3\hat{i} - 6\hat{i} + 4\hat{j}$
$= 3\hat{i} + 4\hat{j}$
$F_{net} = \sqrt{(3)^2 + (4)^2}$
$= 5N$

Inertia: Tendency of a body to resist change in it's state of rest of motion.

LAWS OF MOTION → Newtonian mechanics → Covers the discussion of movement of objects under the influence of forces by making use of Newton's 3 laws.

1. First law
2. Second law
3. Third law

FIRST LAW
Law of inertia
Inertia ∝ mass
Defines force.
↳ Heavier the body, more the inertia...

SECOND
→ Discuss acceleration
→ Calculate mathematical value of F.

THIRD
Forces always act in pair.
$F_{act} = -F_{reac}$

★ If the speeds of interacting objects is very large or comparable to speed of light newtonian mech. is not applicable.

Momentum

Quantity of motion.

$\vec{P} = m\vec{v}$

- Vector Quantity.
- $\vec{P} \parallel \vec{V}$ (Dimensions in direction of velocity)
- Unit = $kgms^{-1}$ or Ns.

MCQ: A ball of mass 'm' is falling in air, rate of change of momentum is:

(a) less than mg (b) equal to mg

Because it's not freely falling & hence friction exists and Fnet = mg − F friction

when body is freely falling...

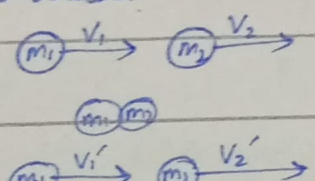
VERY-IMP HINT: When a body is freely falling $a = g$. And bcz $g = 10ms^{-2}$, we know $g = \frac{10ms^{-1}}{1s} = \frac{\Delta V}{\Delta t} = 10ms^{-2}$

From this, we can conclude that with each second the velocity of a freely falling body is changing $10ms^{-1}$. (either increases/decreases).

Now if a ball is thrown downward with $10ms^{-1}$ and after 1 sec it becomes $10ms^{-1}$, hence we conclude it reaches ground in 2s, bcz velocity becomes zero, vice versa for throwing upwards. (velocity adds).

LAW OF CONSERVATION OF MOMENTUM

For isolated system, total momentum remains conserved.



MCQ: Bomb of mass 'm' is at rest, explodes into 2 pieces. One pice mass $m/3$ moves with speed 'v'. Speed of other piece?

$mV = P$ before collision.

$m_1' = m/3 \quad m_2' = 2/3$ (common sense)

$V_1' = V \quad V_2' = ?$

$mV = m_1V_1' + m_2V_2'$

$m(0) = \frac{m}{3}(V) + \frac{2m}{3}(V_2')$

$-\frac{m}{3}V = \frac{2m}{3}(V_2')$

$V_2' = \frac{-3m}{2m}(V) - \frac{m}{3} = \frac{2m}{3}(V_2')$

$= \frac{-6m - 2m}{6m}(V) - \frac{m \times 3}{3 \times 2m} = V_2'$

$\boxed{\frac{-V}{2} = V_2'}$

Second Law of Motion in terms of Momentum:

$\vec{F} = \frac{\Delta P}{\Delta t}$

Impulse: large force in short interval of time.

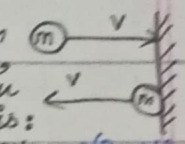
$J/\vec{I} = \vec{F} \times t = \Delta P = mV_f - mV_i$

→ Impulse is a vector quantity in direction of force.

→ Moving body has impulse when it's momentum is changing.

★ Moving body only has impulse when it's momentum is changing.

MCQ: If collision is elastic, it's change in momentum is:



(a) Zero (b) mV (c) −2mV (d) −mV

$(-V) - (V) = -2V$

$\Delta P = m\Delta V = (m)(-2V) = -2mV$

MCQ EXAMPLE: Two Bodies stick together after collision. What is their common velocity?

HINT: either long/short way...

Long: $m_1V_1 + m_2V_2 = (m_1 + m_2)V'$

$V' = \frac{m_1V_1 + m_2V_2}{m_1 + m_2}$

Short: Exactly same, just short by the fact that you can recognize it by hearing.

Common Velocity = $\frac{\text{Total momentum before collision}}{\text{Total mass}}$

GRAPHS

Slope of Graph = $\tan\theta = \frac{\Delta Y}{\Delta X} = \frac{Y_2 - Y_1}{X_2 - X_1}$ → Taken along X-axis

Basic Info:

1. Y, slope = 0, X
2. Y, Slop = ∞, 90°, X
3. Y, slope = const., X
4. Bend hok vertical means increasing slope. ≈ slope = increasing
5. Bend hok horizontal means decreasing slope. ≈ Any graph that bends and become horizontal.
6. Bend ni hoi means slope = const.

MCQ: X, Y, Z
Inc. = Z and Y
Const. = X
Dec. = None
+ve = X and Y
-ve = Z

MCQ: d, A, B, C, D, O, t
Acc = -ve = CD
Velocity = Const. = OA
Velocity = 0 = AB
Acc = +ve = BC

MCQ: A car is moving with a speed of 120 km/h, it's possible graph is:
(a) V, t (b) d, t (c) V, t (d) d, t
(e) c & d

MCQ: Paratrooper jumps from plane it's velocity time graph before opening the parachute:
(a) V, t (b) V, t (c) V, t (d) V, t

Reason: Show v = 0

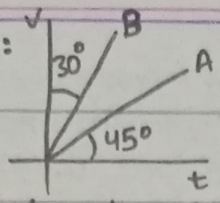
SOME EXAMPLES TO UNDERSTAND

⇒ Hint: Slope = ΔV = a = Same Value

1. V → const. slope = a = ΔV = 0
2. V → increasing slope = a = ΔV = const.
3. V = increase slope = a = ΔV = inc.
4. V = increase slope = a = ΔV = dec.
5. V = decreases slope = a = ΔV = Constant.

ee Decreasing acceleration is not retardation, Decreasing velocity is retardation ee

MCQ:



$\frac{AB}{AA} = ?$

$\frac{AB}{AA} = \frac{\tan(90-30)}{\tan 45} = \frac{\sqrt{3}}{1}$

(a) 1:1 (b) 3:1

(c) $\sqrt{3}$:1 (d) 2:1

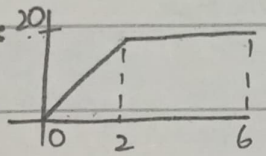
AREA UNDER THE GRAPH

Area under the graph = const. ($\Delta Y \times \Delta X$)

Area under V-t graph = Mag. of Displacement

Area under Speed-t graph = Mag. of Distance

MCQ:



Distance Covered = Area of $\triangle$ + Area of $\square$

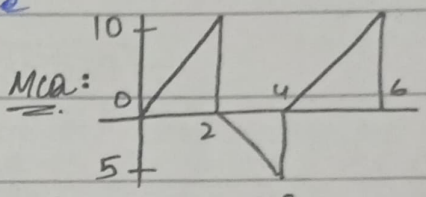
$= \frac{1}{2} \times 20 \times 2 + 4 \times 20$

$= 20 + 80 = 100.$

For first 2 sec = 20

For last 4 sec = 80

MCQ:



Displacement = ?

$\vec{d} = \frac{1}{2} \times 2 \times 10 + \frac{1}{2} \times 2 \times (-5) \times \frac{1}{2} \times 2 \times 10$

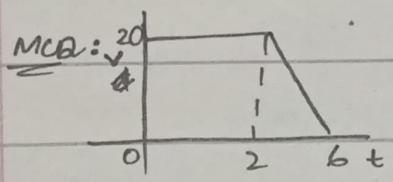
$= 10 + (-5) + 10 = 15\ m$

Distance = ?

$S = \frac{1}{2} \times 2 \times 10 + \frac{1}{2} \times 2 \times 5 + \frac{1}{2} \times 2 \times 10$

$= 10 + 5 + 10 = 25m$

MCQ:



Av. speed = ?

Total dist. $= 20 \times 2 + \frac{1}{2} \times 4 \times 20$

$= 40 + 40 = 80\ m$

Total time = 6s

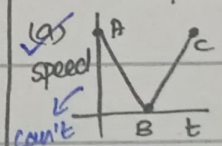
Average Speed = $80/6 = 13.2\ ms^{-1}$

"a" in last 4 sec ?

$a = \text{Slope} = \frac{\Delta Y}{\Delta X} = \frac{0-20}{6-2} = \frac{-20}{4} = -5\ ms^{-2}$

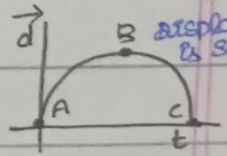
MCQ: A ball is thrown vertically upward neglecting air friction, after reaching the maximum height, it comes back to same point.

(a)



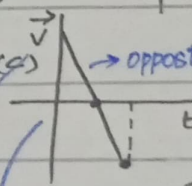
Speed can't be negative

(b) $\vec{d}$



Displacement is same.

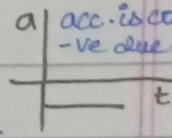
(c)



→ opposite to initial velocity hence graph goes in negative.

slope = -ve = const.

(d)



acc. is const. and -ve due to decreasing velocity

ELASTIC COLLISION

K.E = const.

Momentum const.

T.E = const.

INELASTIC COLLISION

K.E ≠ const.

Momentum const.

T.E = const

* Ball should be hard, smooth & non-rotating.

$v_1' = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 + \left(\frac{2m_1}{m_1 + m_2}\right) v_2$

$v_2' = \left(\frac{2m_1}{m_1 + m_2}\right) v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_2$

CASES OF ELASTIC COLLISION:

CASES	V_1'	V_2'
$m_1 = m_2$	V_2	V_1
$m_1 = m_2$ & $V_2 = 0$	0	V_1
$m_1 >>> m_2$ & $V_2 = 0$	V_1	$2V_1$
$m_1 <<< m_2$ & $V_2 = 0$	$-V_1$	0

MCQ: A proton & neutron moving with $-3ms^{-1}$ and $4ms^{-1}$ respectively show an elastic collision. Their velocities after collision are:
(a) $4ms^{-1}$ & $-3ms^{-1}$
Reason: Not Speed, But velocity interchange means, direction & magnitude both interchange...

PROJECTILE MOTION

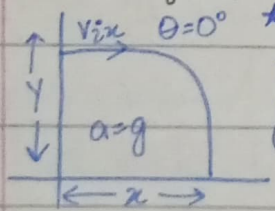
A two dimensional motion in which a body moves with constant horizontal velocity and constant vertical acceleration.

★ Air Friction is neglected.

- Object is thrown in earth's gravitational - field.
- Motion of rockets, crackers, etc. driven by some external force other than gravitational force are not termed as projectile, in general.

HORIZONTAL	VERTICAL
• V_x = constant	• $a_y = g$ = const.
• $V_i = V_f$	• Freely Falling motion
• $a_x = 0$	• F_y = Only weight = Force = $mg = \frac{\Delta P_y}{\Delta t}$
• $\Delta P_x = 0$	• Velocity along the vertical changes constantly.
• P = const.	
• $F = \frac{\Delta P_x}{\Delta t} = 0$	

HORIZONTAL PROJECTILE:



★ Velocity just before landing is more and initial velocity (along x) is less.

i) **Horizontal Distance:**

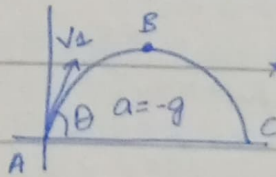
$$S = V_i t + \frac{1}{2} a t^2$$
$$= V_{ix} t + \frac{1}{2} a_x t^2$$
$$= V_{ix} t + \frac{1}{2}(0) t^2$$
$$S = V_{ix} t$$

ii) **Vertical Distance:**

$$S_y = V_{iy} + \frac{1}{2} a_y t^2$$
$$= (0) + \frac{1}{2}(g)(t^2)$$
$$= \frac{1}{2} g t^2$$

OBLIQUE PROJECTILE: (GROUND TO GROUND)

$0° < \theta < 90°$ (along x)



★ $V_{landing} = V_{projection}$ Bcz along x-axis velocity = const.

i) $V_{ix} = V_{fx} = V_i \cos\theta$

ii) $V_{fy} = V_i \sin\theta - gt$

iii) Mag. $V = \sqrt{V_{fx}^2 + V_{fy}^2}$

iv) Direction $= \theta = \tan^{-1}\left(\frac{V_{fy}}{V_{fx}}\right)$

MCQ: A girl throws her engagement ring, from the top of the building of height 10m, if she provides initial velocity along horizontal, $V_{ix} = 20ms^{-1}$ then how far ring will fall?

a) 15m b) 20m

c) 28m d) 32m

Time Calculation $= Y = \frac{1}{2}gt^2$

$t^2 = \frac{2Y}{g}$

$t = \sqrt{\frac{2Y}{g}}$

$t = \sqrt{\frac{2\times10}{10}}$

Ring remains in air for $\rightarrow$ $t = \sqrt{2} = 1.45$

$V_{ix} \times t$

$X = (20)(1.4) = 28m$

MCQ: A girl throws her engagement ring, from top of the building, if she provides $V_{ix} = 30ms^{-1}$, then after 4s V_2?

(a) $30ms^{-1}$ (b) $50ms^{-1}$

(c) $70ms^{-1}$ (d) $60ms^{-1}$

$V_{fy} = V_i\sin\theta - gt$

$= V_i(\sin\theta) - gt \therefore \theta = 0°$ for horizontal projectile

$V_{fy} = -gt = (10)(4) = 40$

$V = \sqrt{(30)^2 + (-40)^2}$

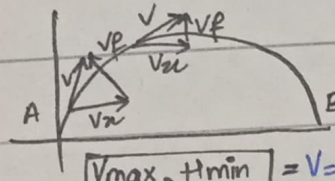
$= \sqrt{900+1600}$

$= \sqrt{2500}$

$= 50ms^{-1}$

(OR)

As we know for freely falling 1sec $\rightarrow$ change 10 ms^{-1} with each sec, 10ms^{-1} changes, isse time hm bagair formula bagair time calculate kr sktyhy...



$V_{max}, H_{min} = V = \sqrt{V_{fx}^2 + V_{fy}^2}$

$V = \sqrt{V_{fx}^2 + V_{fy}^2} = V_{min}, H_{max}$

$V = \sqrt{V_{fx}^2 + 0}$

$V = V_{fx}$

$V_{H\text{-}max} = V_{fx} = V_i\cos\theta = V_{min}$ (V_i = V_{max})

* In projectile motion, velocity of an object can never be zero.
* Hence, K.E can never be completely converted to P.E, if this happens it means K.E = 0 and for that $V = 0$, whereas in projectile motion V can never be 0.

IMP FORMULA:

$V = \sqrt{V_{fx}^2 + V_{fy}^2}$

$= \sqrt{(V_i\cos\theta)^2 + (V_i\sin\theta - gt)^2}$

$= \sqrt{V_i^2\cos^2\theta + V_i^2\sin^2\theta + g^2t^2 - 2V_i\sin\theta gt}$

$= \sqrt{V_i^2(\cos^2\theta + \sin^2\theta) + g^2t^2 - 2V_i\sin\theta gt}$

$= \sqrt{V_i^2(1) + g^2t^2 - 2V_i\sin\theta gt}$

$V = \sqrt{V_i^2 + g^2t^2 - 2V_i\sin\theta gt}$

$V_{H\text{-}MAX} = V_i\cos\theta$

$\theta = 30°$	$\theta = 45°$	$\theta = 60°$
$\frac{\sqrt{3}}{2}V_i$	$\frac{1}{\sqrt{2}}V_i$	$\frac{1}{2}V_i$

$P_{H\text{-}MAX} = P_i\cos\theta$

$\theta = 30°$	$\theta = 45°$	$\theta = 60°$
$\frac{\sqrt{3}}{2}P_i$	$\frac{1}{\sqrt{2}}P_i$	$\frac{1}{2}P_i$

$K.E_{H\text{-}MAX} = KE_i\cos^2\theta$

$\theta = 30°$	$\theta = 45°$	$\theta = 60°$
$K.E_{H\text{-}max} = \frac{3}{4}K.E_i$	$\frac{1}{2}K.E_i$	$\frac{1}{4}K.E_i$
$(\sqrt{3}/2)^2$	$(1/\sqrt{2})^2$	$(1/2)^2$

$P.E_{H\text{-}MAX} = K.E_i\sin^2\theta$

$\theta = 30°$	$\theta = 45°$	$\theta = 60°$
$\frac{1}{4}K.E_i$	$\frac{1}{2}K.E_i$	$\frac{3}{4}K.E_i$

* "Sin" dependant increase values with increase in angle and vice versa for cos values.

∴ Angle along x-axis is 90°

MCQ: A football player kicked a ball having total energy 200J. For which angle of projection P.E at highest point becomes 50J.

(a) $\theta = 30°$ (b) 60°

(c) 45° (d) None of these

Sol: Total Energy = Max Energy = $K.E_i = 200J$

$P.E_{H\text{-}max} = K.E_i \sin^2\theta$

$= 200(\sin^2 30)$

$= (200)(1/4)$

$= 50J$

$\therefore \theta = 30° = 1/4$

MCQ: A player kicked a football with 60ms^{-1} at an angle 60° with Y-axis. It's velocity at H max?

(a) 51ms^{-1}

$V_x = V_x = V_i \cos\theta$

$= (60)\cos(90-60) = 60\cos 30$

$= 60 \times \frac{\sqrt{3}}{2} = 30 \times \sqrt{3}$

$= 30 \times 1.7$

$V = 51 ms^{-1}$

MCQ: Two balls A and B are projected with same initial velocity by making an angle 30° with x-axis and 30° with Y-axis. Ratio of their heights?

(a) 3:1 (b) 1:3 (c) 1:1 (d) 1:4

$\theta_Y = 30°$

$= 90 - 30°$

$= 60°$

$\frac{H_A}{H_B} = \frac{(\sin 30)^2}{(\sin 60)^2} = \frac{1/4}{3/4} = \frac{1}{3} = 1:3$

HEIGHT OF PROJECTILE

Max. vertical distance from height to the ground.

$H = \frac{V_i^2 \sin^2\theta}{2g}$ (OR) $H = \frac{V_{iy}^2}{2g}$

Depends: Initial velocity, $\sin^2\theta$

$\alpha \frac{1}{g}$

MCQ: For which angle of the projection, height of projectile will be max, if initial velocity is same?

(a) 30° (b) 0°

(c) 60° (d) 85°

The value nearest to 90° will give max. value of H.

TIME OF FLIGHT

i) $T = \frac{2V_i \sin\theta}{g} = \frac{2V_{iy}}{g}$

$T \alpha V_i$ $T \alpha \sin\theta$ $T \alpha \frac{1}{g}$

ii) Time to reach max Height:

$T_{(H\text{-}max)} = \frac{T}{2} = \frac{V_i \sin\theta}{g} = \frac{V_{iy}}{g}$

MCQ: A footballer kicked a ball with 10ms^{-1} by making an angle 30° with x-axis. In how many secs, ball will reach max height?

$T_{(H\text{-}max)} = \frac{10 \sin(90)}{10} = \frac{1}{2} = 0.5s$

How many secs ball will remain in the air?

$T = \frac{2V_{iy}}{g} = \frac{2(10)\sin 30}{10} = \frac{2 \times 1}{2} = 1s$

MCQ: 2 balls A and B are projected with same initial velocity by making an angle θ and $90-\theta$ with x-axis, the ratio of their time of flight is:

(a) $\tan\theta : 1$ (b) $1 : \tan\theta$

(c) 1:1 (d) $\sin\theta : 1$

HINT: $\sin(90 \pm \theta) = \cos\theta$

$\cos(90 \pm \theta) = \sin\theta$

$\frac{T_A}{T_B} = \frac{\sin\theta}{\sin(90-\theta)} = \frac{\sin\theta}{\cos\theta} = \frac{\tan\theta}{1}$

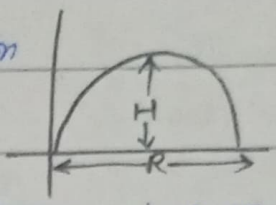
$\tan\theta : 1$

$\therefore \frac{\sin\theta}{\cos\theta} = \tan\theta$

RANGE

Horizontal distance from point of projection to the point of landing.

$$R = \frac{V_i^2 \sin 2\theta}{g}$$



ii) Max. Range $= \frac{V_i^2}{g}$

iii) $R = (R_{max})(2\sin\theta)$

For Range (max) $\sin 2\theta$ must be equal to 90°:

$\sin 2(45) = \sin 90 = 1$

MCQ: A footballer kicked a ball (horiz) covering max. distance of 10 m. The velocity with which ball is projected is?

(a) 5m (b) 10m ✓ $R_{max} = \frac{V_i^2}{g}$

(c) 50m (d) 15m

MCQ: For which angle of projection, range of projectile will be maximum?

(a) 50° (b) 60° (Nearest to 45°)

✓ (c) 47° (d) 30°

MCQ: For which angle of projection, range of projectile becomes half of it's maximum value?

✓ (a) 75° (b) 30° (c) 22.5° (d) 45°

Sol: 90 − 75 = 15°

Range will be same as it will be on 15°. Infact any 2 angles giving sum of 90° have same range.

SAME RANGE

30, 60
15, 75
45, 45
40, 50
20, 70

We know, $\sin 30 = 1/2$

But for $2\sin\theta = 2\sin(15)°$

MCQ: $R_{MAX} = 40$, $H = ?$

(a) 40m (b) 10m

(c) 160m (d) 80m

$R_{max} = \frac{V_i^2}{g}$

$V_i^2 = (R_{max}) \times g$

$= 40 \times 10$

$V_i^2 = 400$

$V_i = 20$

$H = \frac{V_{iy}^2}{2g} = \frac{(20)^2}{2 \times 10}$

$H = 20m$

Whereas,

$H = \frac{R_{max}}{4}$

$H = \frac{40}{4} = 10m$

$H = 10m$

Short-Cut:

$R\tan\theta = 4H$

$\theta < 76°$	$\theta = 76°$	$\theta > 76°$
$R > H$	$R = H$	$R < H$

$R\tan\theta = 4H$

Prove $R = H$

So, $(R/H)\tan\theta = 4H$

$H\tan\theta = 4H$

$\tan\theta = 4$

$\theta = \tan^{-1} 4$

$\theta = 76°$

$R_{max} = 4H$

$\text{Range} = V_i^2 \times \sin 2\theta / g$

$\therefore \sin 2\theta = 2\sin\theta\cos\theta$

$\text{Range} = \frac{V_i^2 \times 2\sin\theta\cos\theta}{g}$

$= \frac{2V_i\sin\theta \times V_i\cos\theta}{g}$

$= \frac{2V_{ix} \times V_{iy}}{g}$

Relations:

$R_{max} = 4H$

B/w Time of Flight & Height $= H = \frac{gt^2}{8}$

B/w Time of Flight & $R(max) = \frac{R_{max}}{4} = \frac{gt^2}{8}$

$\therefore R_{max} = \frac{gt^2}{8} \times 4$

$\therefore R_{max} = \frac{gt^2}{2}$